this time: Bayes's Theorem

next time: random variables

read: DS ch. ③

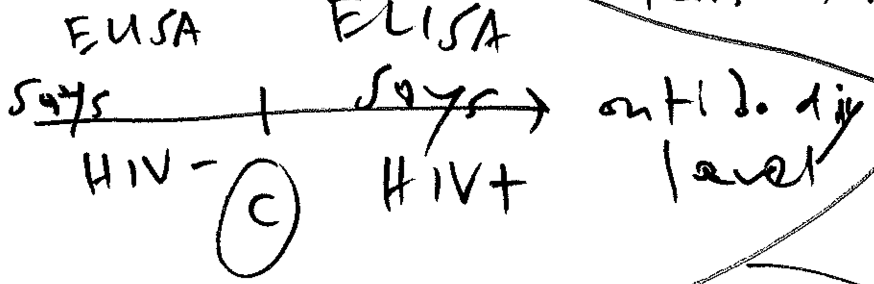
AMS131
2 Jul 18

①

no class or office hours wed 4 July ← big setback for us finishing all the required material

will probably need a make-up lecture (webcast)

take-home test 1 not due this Fri (due sometime next week)



$P(A) = 0.4\% = 0.004$

prevalence

sensitivity

$P(+ \mid A) = 96\% = 0.96$

specificity

$P(- \mid \text{not } A) = 97\% = 0.97$

method 1

truth

sensitivity

		blood has HIV (A)	blood doesn't have HIV $= A^c$ (not A)	
ELISA says	+	384	2,988	3,372
	−	16	96,612	96,628
		400	99,600	100,000

specificity

prevalence

$P(\text{not } A \mid -) = \frac{96612}{96628} = 0.9998 \doteq 99.98\%$

ELISA excellent when ⊖

$$p(A|+) = \frac{384}{3372} \doteq 0.11 \doteq 11\% \text{ (!)}$$

false rate 89%

ELISA is crap when ⊕

when is $\frac{a}{b}$ large ?

when ⓐ is large or ⓑ is small or both

384 is small because HIV prevalence is low

---

doesn't help much to increase ELISA's sensitivity

to improve $p(A|+)$, need to increase specificity

truth

		A	not A
ELISA	+	✓	X
	-	X	✓

blood not HIV+ but ELISA says ⊕ (false positive)

throw blood unit away (lose blood)

blood is HIV+ but ELISA says ⊖ (false negative)

← some innocent person is given HIV (terrible)

③ method

③

$$P(A \mid +) = \frac{\boxed{P(A)}\, P(+ \mid A)}{\boxed{P(+)}}$$

unknown ↗ ↖ data

(hard) annoying normalizing constant

$P(A) = 0.004$

(prior)

$P(+ \mid A) = 0.96$

sensitivity

(likelihood)

method ②

$$P(A \mid +) = \frac{P(A)\, P(+ \mid A)}{P(+)}$$

divide

$$P(\text{not } A \mid +) = \frac{P(\text{not } A)\, P(+ \mid \text{not } A)}{P(+)}$$

$$\left(\frac{P(A \mid +)}{P(\text{not } A \mid +)}\right) = \left(\frac{P(A)}{P(\text{not } A)}\right) \cdot \left(\frac{P(+ \mid A)}{P(+ \mid \text{not } A)}\right)$$

(posterior odds ratio in favor of A given data (+)) = (prior odds ratio in favor of A) (likelihood ratio ≜ Bayes factor)

Bayes's Theorem in odds form

$P(A) = p$

$P(\text{not } A) = 1 - p$

$$\frac{P(A)}{P(\text{not } A)} = \frac{p}{1-p} = \text{odds ratio in favor of } A$$

sensitivity (+)

$$\frac{P(A|+)}{P(\text{not } A|+)} = \left(\frac{0.004}{0.996}\right)\left(\frac{0.96}{1-0.97}\right)$$

specificity

$$\boxed{7.78}$$

90

[illegible]

$$\left(\begin{array}{c}\dfrac{249}{32}\\ \text{odds}\\ \text{against}\\ A\end{array}\right) = \left(\begin{array}{c}249\\ \text{to } 1\\ \text{odds}\\ \text{against}\\ A\end{array}\right) \cdot \left(\begin{array}{c}32\\ \text{to } 1\\ \text{data odds}\\ \text{in favor}\\ \text{of } A\end{array}\right)$$

$$\text{if } \frac{p}{1-p} = \frac{P(A)}{P(\text{not } A)} = \text{odds ratio in favor of } A$$

$$\text{then } \frac{P(\text{not} A)}{P(A)} = \frac{1-p}{p} = \text{odds ratio against } A$$

$$\text{if } \underset{\substack{\uparrow \\ \text{odds ratio}}}{o} = \frac{p}{1-p} \longleftrightarrow p = \frac{o}{o+1}$$

$$\frac{\boxed{P(A|+)} \leftarrow p}{P(\text{not } A|+)} = \frac{32}{249} = o \leftrightarrow p = \frac{o}{1+o}$$

method ③

$$P(A|+) = \frac{\overset{\checkmark}{P(A)}\cdot \overset{\checkmark}{P(+|A)}}{\boxed{P(+)}}$$

$$= \frac{\frac{32}{249}}{1+\frac{32}{249}} = \frac{32}{281} = \left(\frac{384}{3372}\right) = 0.11 \checkmark$$

$P(+) = ?$ extend the conversation to include truth

(hard)

⊕ depends on A

$$P(+) = P(+ \text{ and } A) + P(+ \text{ and not } A)$$

$$= P(A)\cdot P(+|A) + P(\text{not } A)\cdot P(+|\text{not } A)$$

$$P(A|+) = \frac{P(A)\cdot P(+|A)}{P(A)\cdot P(+|A) + P(\text{not } A)\cdot P(+|\text{not } A)}$$

so $$P(A|+) = \frac{(0.004)(0.96)}{(0.004)(0.96) + (0.996)(1-0.97)}$$

$$= \frac{384}{3372} = 0.11 \checkmark$$

(10.42)