

this time: permutations & combinations

read: DS ch. 1

AMS 131  
27 Jun 18

next time: axioms

$$P(\text{inf in } n \text{ acts}) = \textcircled{1}$$

$$P(\text{inf on set 1} \textcircled{\text{or}} \text{inf on set 2} \textcircled{\text{or}} \dots \text{or inf on set } n) \textcircled{\text{hard}}$$

$$P(A \text{ or } B) = 1 - P(\text{not inf. in } n \text{ acts})$$

$$P(A) + P(B) - P(A \text{ and } B) = 1 - P(\text{not inf. on set 1} \textcircled{\text{and}} \text{not inf. on set 2})$$

$$P(\text{10 or more TS babies in 5 kids}) \textcircled{\text{IID}} = 1 - P(\text{not inf. on 1st} \textcircled{\text{and}} \text{not inf. on 2nd}) \dots P(\text{not inf. on } n \text{th})$$

$$P(A \text{ and } B) =$$

$$P(A) \cdot P(B|A)$$

$$P(B) \cdot P(A|B)$$

if A, B indep:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$\text{IID} = 1 - \left(1 - \frac{1}{500}\right) \left(1 - \frac{1}{500}\right) \dots \left(1 - \frac{1}{500}\right) \quad (2)$$

$$= 1 - \left(1 - \frac{1}{500}\right)^n = 1 - (1 - p)^n,$$

$$p = \frac{1}{500}$$

	h	Schram	right
	:		
	n	$n p$	$1 - (1 - p)^n$
	1	p	p
$p = \frac{1}{500}$	100	0.2	$1 - \left(1 - \frac{1}{500}\right)^{100} \approx 0.18$
	500	1.0	$1 - \left(1 - \frac{1}{500}\right)^{500} \approx 0.63$
	600	1.2 (!)	$1 - \left(1 - \frac{1}{500}\right)^{600}$

n	Schwarz	right
2	$2p$	$1 - (1-p)^2 =$ $1 - [1 - 2p + p^2]$ $= 2p - p^2$
n	np	$1 - (1-p)^n = np$ - (positive stuff)

causal inference → how prove that A causes B? (cause effect)

before 1750 → deterministic } causality  
 probabilistic }

(Bayes 1750) → possible causes:
 

- water
- food
- air
- disease

 village  
 some people dying } effect

$P(E|C)$   
↑  
effect

↑  
cause

how related to

$P(C|E)$ ?  
harder

(4)

easier

~~$P(A|B) = P(B|A)$~~   
always?

$P(\text{rain on you} | \text{clouds above you})$

low

$P(\text{clouds above you} | \text{rain on you})$

= high

~1920

Fisher

controlled

experiments

treatment (T)  
&  
control (C)  
groups

← experimentally assign subjects to (T) & (C)

outcome (effect) (I)

supposedly causal factor (X) →

experimentally manipulate

goal want (T), (C)

subjects as similar as possible in all relevant ways, except for (T)/(C) distinction

method assign subjects to (T), (C) (5)

at random

randomized controlled

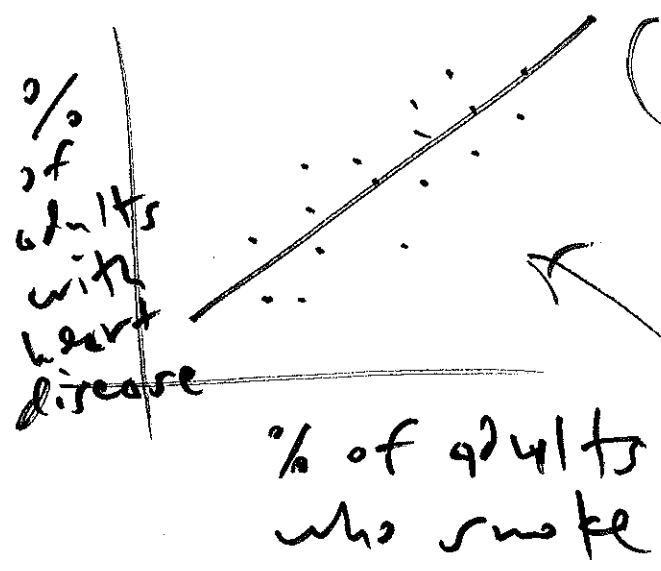
~~or~~ trial

can't run RCT (ethics)

(RCT)

outcome: lung cancer or not (Y)

SCF (X):  
smoke (T)  
not smoke (C)



(1958)

epidemiologist  
K. Doll

1 dot for each country

Constitutional

hypothesis (CH)

$$P(\text{both smokers died first of lung cancer}) = P(HH) \quad \text{fair} \quad (6)$$

$$= P\left(\begin{array}{c} H \\ 0.5 \\ 1^{st} \end{array} \text{ and } \begin{array}{c} H \\ 0.5 \\ 2^{nd} \end{array}\right)$$

$$P(\text{all 9 smokers died first of heart disease}) = P\left(\begin{array}{c} H \\ 0.5 \\ 1^{st} \end{array}\right) \cdot P\left(\begin{array}{c} H \\ 0.5 \\ 2^{nd} \end{array}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(HH \dots H) = \left(\frac{1}{2}\right)^9 = \frac{1}{512}$$

① temporarily pretend

$$= 0.002$$

$$= 0.2\%$$

CA is right

② under ①, data set is

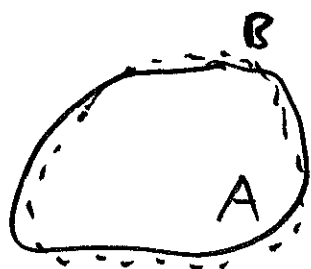
prob. version of proof by

contra dictio

extremely unlikely

③  $\therefore$  temporary pretend in ① is probably wrong

(10.07)



if  $A = B \rightarrow$

$$P_K(A) = P_K(B)$$

(continuity)

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equivalent to  $\textcircled{K}$  axiom 3



if  $A_n \rightarrow B$  then

$$P(A_n) \rightarrow P(B)$$

continuity in the limit

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