Discussion Section 5

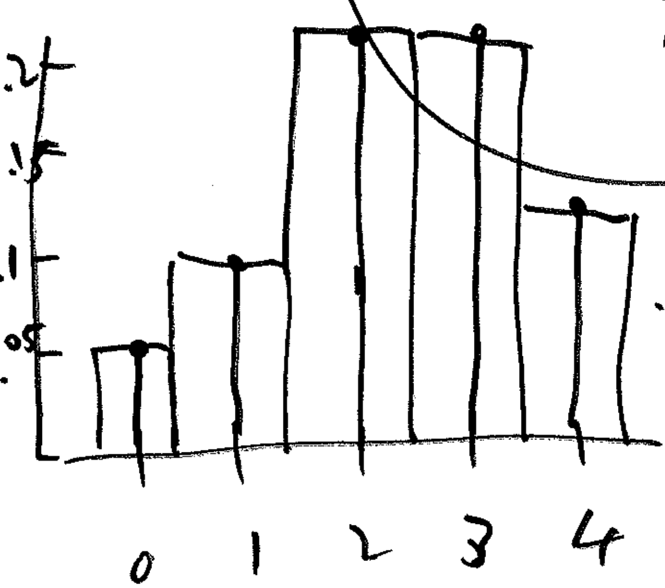
R exploring the Poisson distribution ← Siméon Poisson (1837)

$(\lambda > 0)$

$(Y \mid \lambda) \sim \text{Poisson}(\lambda) \longleftrightarrow$ (PMF)

$$f_Y(y) = \begin{cases} \dfrac{\lambda^y e^{-\lambda}}{y!} & \text{for } y = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

$$= \frac{\lambda^y e^{-\lambda}}{y!} \, I_{\underbrace{\{0, 1, \ldots\}}_{\text{support}}}(y)$$



dpois (PMF)

ppois (CDF)

qpois (inverse CDF)

rpois simulating

$\lambda = 3$

$\lambda = 0.5$ ← reverse J-shape

DS. p.141 #8

X, Y continuous RVs

joint PDF

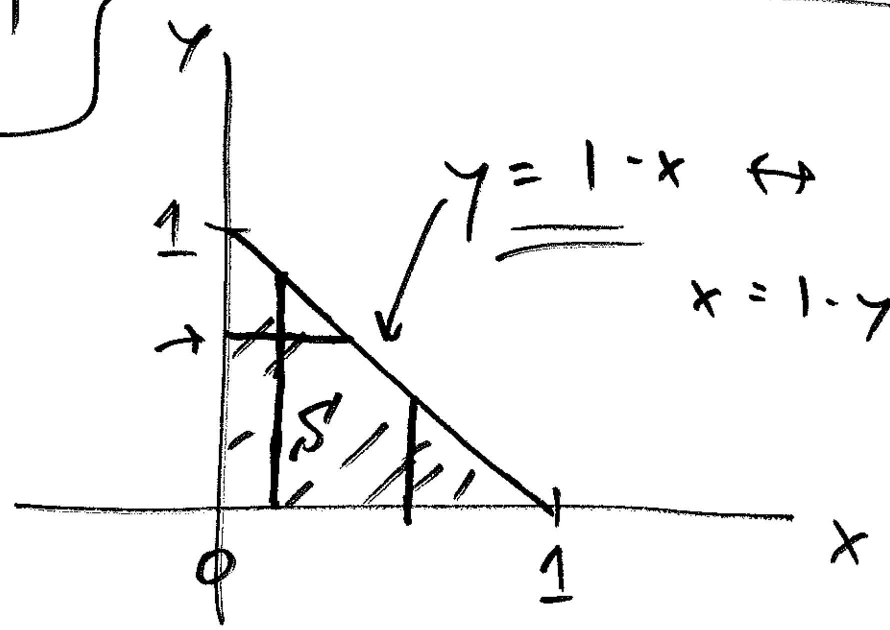
$$f_{XY}(x,y) = \begin{cases} 24xy & \text{for } \left\{ \begin{matrix} x \ge 0, & y \ge 0 \\ x+y \le 1 \end{matrix} \right\} \\ 0 & \text{else} \end{cases}$$

(b) verify that 24 is the correct normalizing constant

(a) sketch the support S of this PDF

$x + y \le 1 \leftrightarrow$

$y \le 1 - x$



show

$$1 = \iint_S f_{XY}(x,y)\,dx\,dy$$

$$= \int_0^1 \int_0^{1-x} 24xy\,dy\,dx = 24 \int_0^1 x \left[\int_0^{1-x} y\,dy \right] dx$$

$$= 24\int_0^1 x\left[\frac{y^2}{2}\right]_0^{1-x} dx = 24\int_0^1 x\,\frac{(1-x)^2}{2}\,dx$$

$$\boxed{= \int_0^1\int_0^{1-y} f_{XY}\,dx\,dy}$$

$$= \frac{24}{2}\int_0^1 x\left(1-2x+x^2\right)dx$$

$$= 12\int_0^1 \left(x-2x^2+x^3\right)dx$$

$$\therefore = 1$$

(c) work out $f_X(x)$ for $0 \le x \le 1$

$$f_X(x) = \int_S f_{XY}(x,y)\,dy = \int_0^{1-x} 24xy\,dy$$

to get the marginal for X, integrate out Y, & vice versa

$$f_X(x) = \int_0^{1-x} 24xy\,dy = 24x\int_0^{1-x} y\,dy$$

$$= 24x\left[\frac{y^2}{2}\right]_0^{1-x} = \frac{24x(1-x)^2}{2}$$

$$= 12x(1-x)^2$$

for $0 \le x \le 1$

to be a PDF

$$\int_0^1 12x(1-x)^2\,dx = 1$$

(d) work out $f_Y(y)$ for $0 \le y \le 1$ (4)

$$f_Y(y) = \int f_{XY}(x,y)\,dx$$

$$= \int_0^{1-y} 24xy\,dx = 12y(1-y)^2$$

for $0 \le y \le 1$

(e) are X & Y independent?

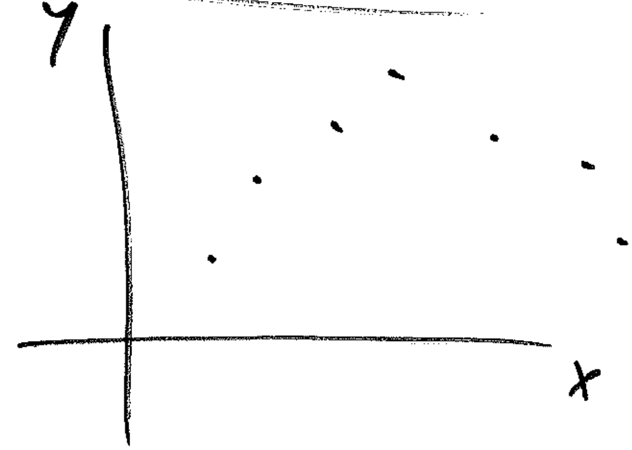
no, they're dependent

$$f_{XY}(x,y) \overset{?}{\neq} f_X(x) \cdot f_Y(y)$$

$$24xy \text{ for } \left\{ \begin{array}{l} x \ge 0,\ y \ge 0 \\ x+y \le 1 \end{array} \right\}$$

$$f_X(x) = 12x(1-x)^2$$

$$f_Y(y) = 12y(1-y)^2$$



plot(x,y)

A, B T/F prop

P(A and B) = P(A) · P(B) iff A, B indep.

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

A, B T/F prop.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

if $P(A) > 0$

$$= \begin{cases} \dfrac{24xy}{12x(1-x)^2} & \text{for } x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0 & \text{else} \end{cases}$$

$$\frac{2y}{(1-x)^2}$$

$$f_{X|Y}(x|y) = \begin{cases} \dfrac{2x}{(1-y)^2} & x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0 & \text{else} \end{cases}$$

~~T/F prop.~~

A, B indep. $\longleftrightarrow$ $P(A|B) = P(A)$

$P(B|A) = P(B)$

$\overset{A}{X}, \overset{B}{Y}$ indep $\Longleftrightarrow f_{X|Y}(x, y) = f_X(x)$

& $f_{Y|X}(\cancel{y}x, y) = f_Y(y)$