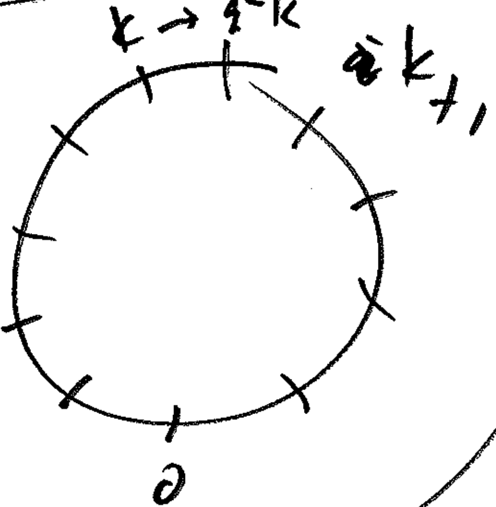
this time: Markov chains

extra credit quiz 10 now available

AMS131
27Jul18

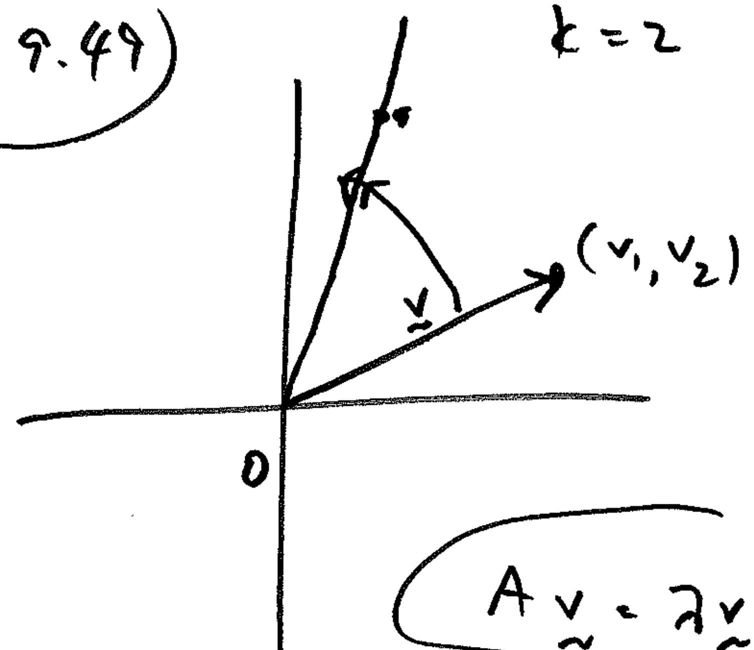
①

social office hour Mon 30 Jul 4pm → Lúpulo (downtown Santa Cruz)



9.49)

$k = 2$



$$A_{k\,k} = A_{2\,2}$$

$\det(A) \neq 0$

(A full rank)

$$\underset{k\,k}{A}\ \underset{k\sim 1}{v}$$

$$A \underset{\sim}{v} = \lambda \underset{\sim}{v}, \quad \lambda \neq 0$$

(10.26)

$$\bar{D}_n = \frac{1}{n}\sum_{i=1}^{n} D_i$$

IID

$N(\Delta, \sigma_D^2)$

$$E(\bar{D}_n) = \Delta \qquad V(\bar{D}_n) = \frac{\sigma_D^2}{n}$$

standard error

$$SD(\bar{D}_n) = \sqrt{V(\bar{D}_n)} = \frac{\sigma_D}{\sqrt{n}} = SE(\bar{D}_n)$$

pop.

all hypertensive uk adults in 1975

B_i A_i D_i

$N = ?$ (big)

mean $\Delta = ?$

SD $\sigma_D = ?$

sample

the observed patients

before after

(i) person	B_i	A_i	$D_i = B_i - A_i$
1	200	191	
2			
12	210	177	

like IID

$n = 12$

(mmHg systolic)

mean $\bar{D}_n = \hat{\Delta} = \hat{\Delta}_n = 18.6$

SD $s_D = 10.1$

sample histogram

$\underset{\sim}{D} = (D_1, \ldots, D_n)$

Given this dataset & little or no info. about Δ external to $\underset{\sim}{D}$, we would conclude (infer) that Δ is around $\underline{\bar{D}_n = 18.6}$, give or take $\underline{2.9}$, (*)

estimated $SE(\bar{D}_n) = \hat{SE}(\bar{D}_n) = \frac{\cancel{\sigma}\, s_D}{\sqrt{n}}$

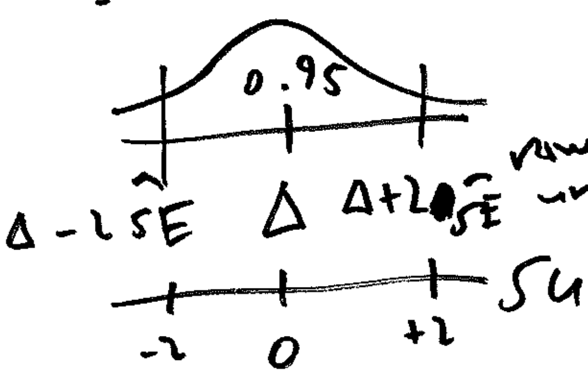
$= \frac{10.1 \text{ mmHg}}{\sqrt{12}} = 2.92 \doteq 2.9 \text{ mmHg}$

~~(Jerry)~~ ~~(frequentist)~~

Jerzy Neyman (- 1981)

$\hat{SE}$ 2.9

PDF of $\bar{D}_n$



95% confidence interval for Δ (CI)

$\bar{D} - 2\hat{SE}$ $\bar{D}_n$ $\bar{D}_n + 2\hat{SE}$

$\Delta \geq \bar{D}_n - 2\hat{SE}$

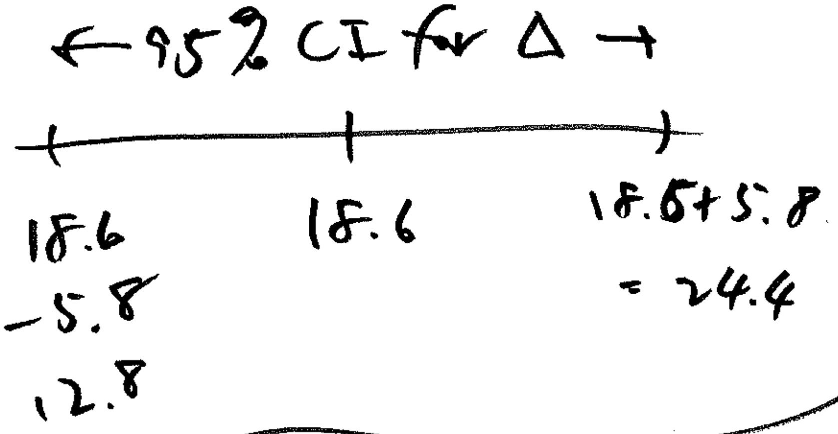
$$P(\Delta - 2\hat{SE} \leq \bar{D}_n \leq \Delta + 2\hat{SE}) \doteq 0.95$$

$\Delta \leq \bar{D}_n + 2\hat{SE}$

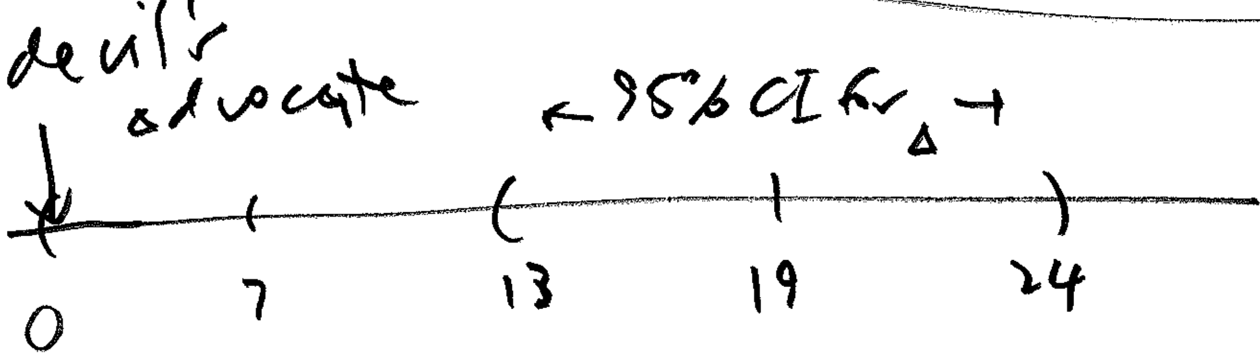
$$P(\bar{D}_n - 2\hat{SE} \leq \Delta \leq \bar{D}_n + 2\hat{SE}) \doteq 0.95$$

random — fixed unknown number — random

Mr. Neyman proposed $\bar{D}_n \pm 2\hat{SE}(\bar{D}_n)$ as an approximate 95% confidence interval for Δ



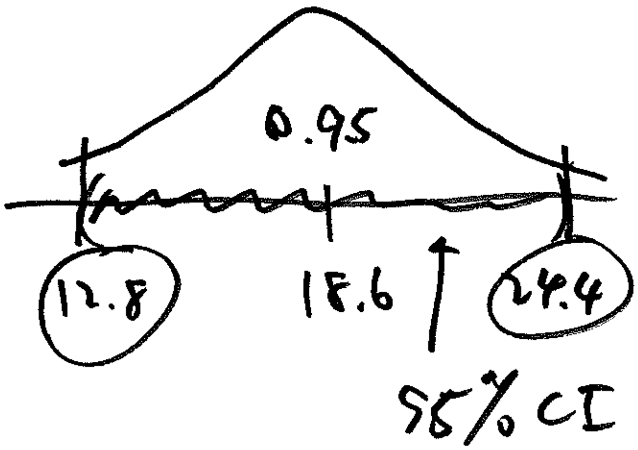
and we're pretty sure (95% confident) that Δ is between 12.8 and 24.4



nothing

since $\Delta_0 = 0$ (null value) is not inside the 95% CI, the diff. between 0 & $\bar{D}_n = 18.6$ is statistically significant

$\longleftrightarrow$ difficult to attribute to unlucky random sampling $\longleftrightarrow$ probably real

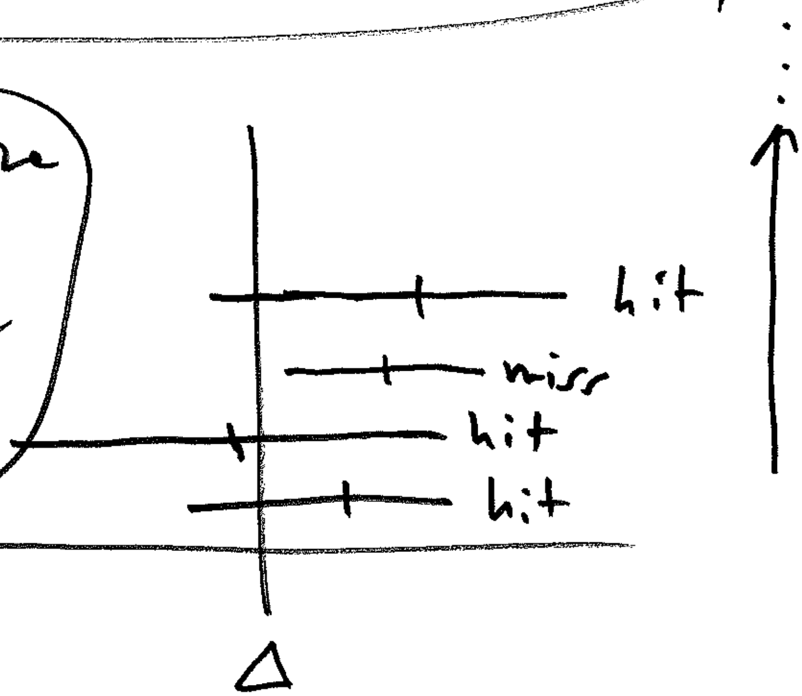


$p(\Delta \mid \underset{\sim}{D}, \text{little external info})$

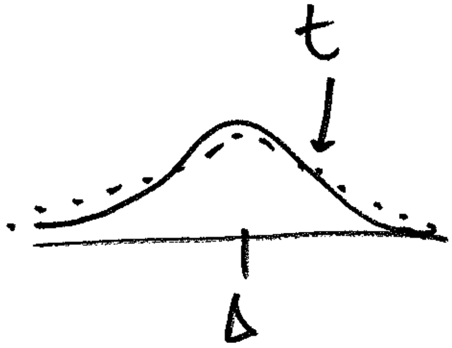
$P_F(12.8 \le \Delta \le 24.4)$?
$\doteq$ ~~0.95~~
undefined

confidence ≠ probability

95% of the intervals would be hits



your confidence is in the <u>process</u> of building the CI, not in the <u>outcome</u> from any single sample we cheated by pretending $\sigma_D = s_D$



PDF of $\bar{D}_n$, accounting for uncertainty about σ_D

William Gosset (1908) (Guinness Brewery)