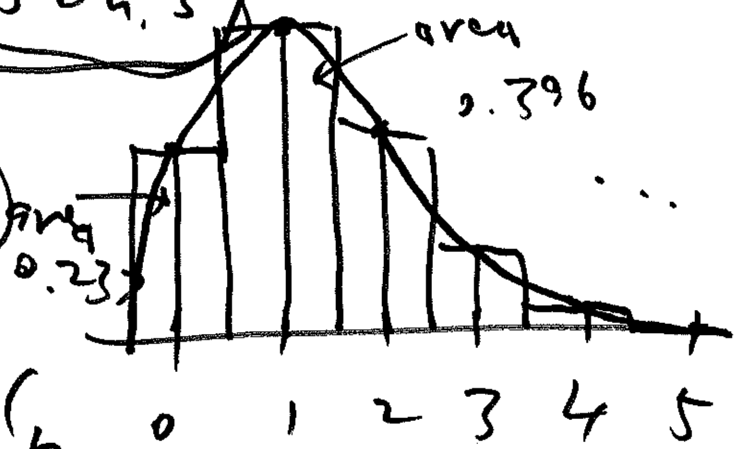
this time: densities & CDFs

read: (DS ch. 1, 2) DS ch. 3

next time 2 or more RVs at a time



PMF of Binomial$(n=5,\ p=\frac{1}{4})$

$$f_Y(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y} & \text{for } y = 0, 1, \dots, 5 \\ 0 & \text{else} \end{cases}$$

$Y \sim \text{Binomial}(5, \frac{1}{4})$ — n, p

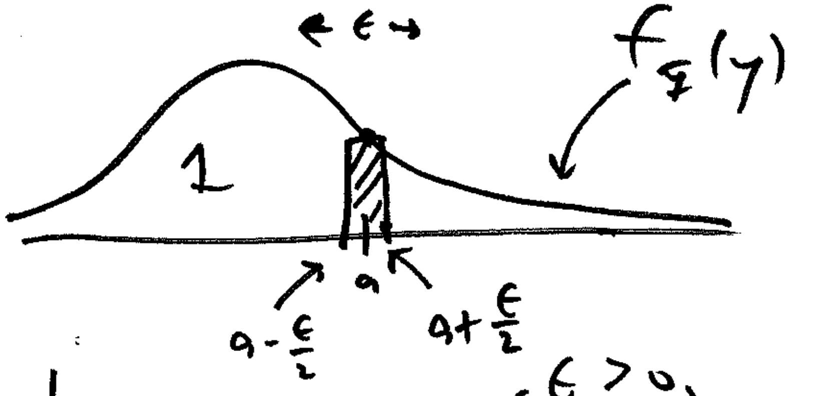
T-s babies

$$\sum_{y=0}^{5} f_Y(y) = 1$$

$$\left(\begin{array}{c}\text{sum of}\\ \text{areas of}\\ \text{6 bars}\end{array}\right) = 1 \doteq \left(\begin{array}{c}\text{area under}\\ \text{approximation}\\ \text{curve}\end{array}\right)$$

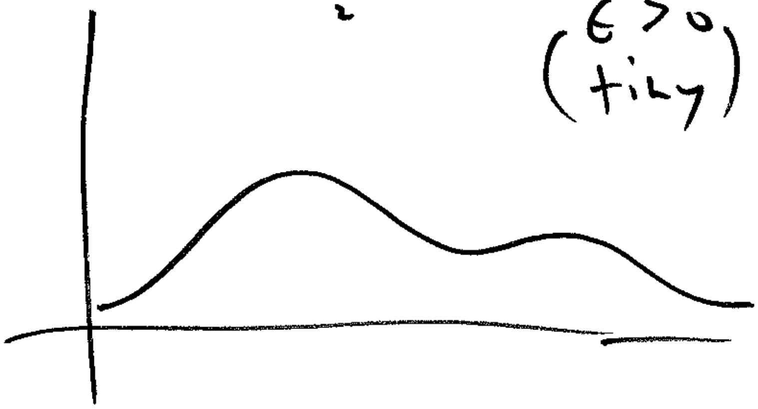
probability = area under a curve from one place to another

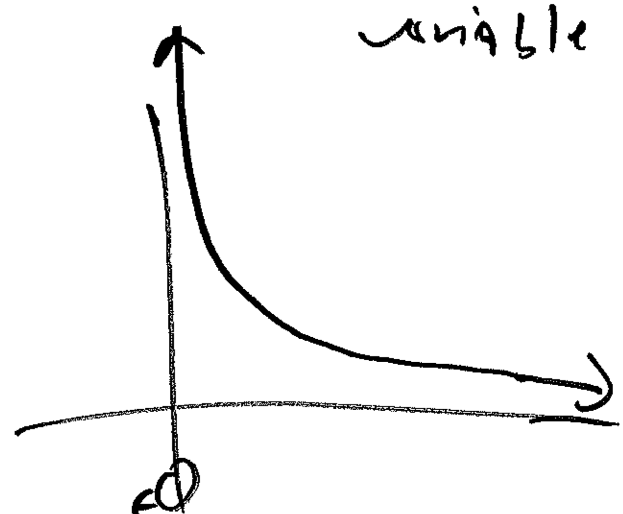
(total area under curve = 1)



Y continuous RV (random variable)

($\epsilon > 0$ tiny)





$$P\left(a-\frac{\epsilon}{2} \le Y \le a+\frac{\epsilon}{2}\right) = \int_{a-\frac{\epsilon}{2}}^{a+\frac{\epsilon}{2}} f_Y(y)\,dy$$

$$\doteq (\text{base})(\text{height}) = \epsilon\, f_Y(a)$$

$$f_Y(a) \doteq \frac{P\left(a-\frac{\epsilon}{2} \le Y \le a+\frac{\epsilon}{2}\right)}{\epsilon}$$

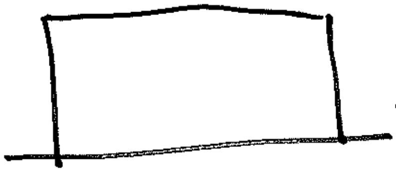
density

= probability per (unit) width

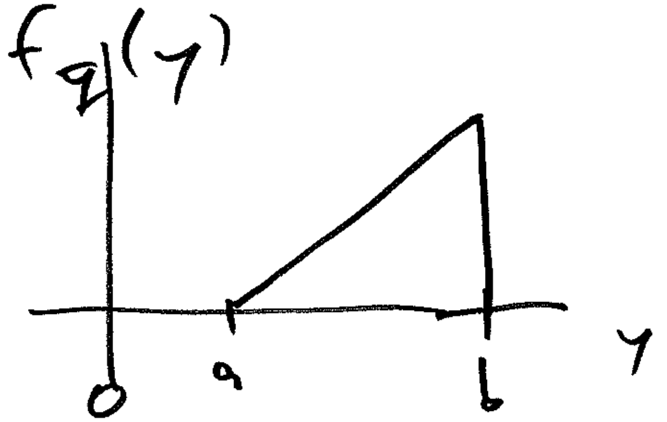
human population density = # people per square mile

$$f_Y(a) = \lim_{\epsilon \to 0} \frac{P\left(a - \frac{\epsilon}{2} \le Y \le a + \frac{\epsilon}{2}\right)}{\epsilon}$$

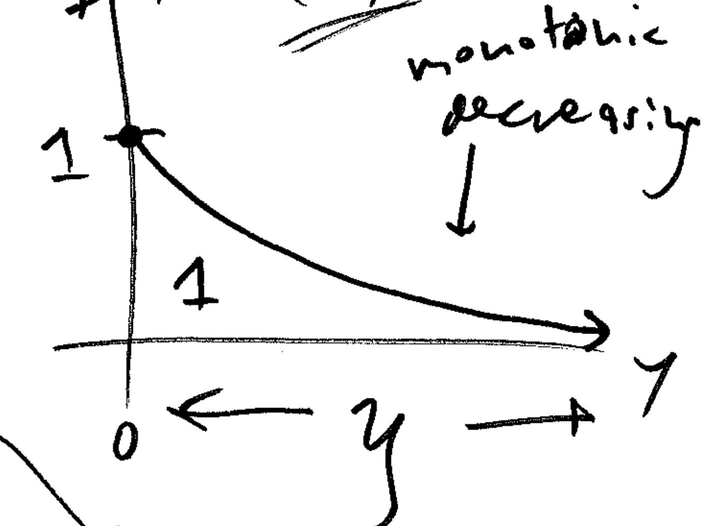
Q: can $f_Y(y)$ be constant?

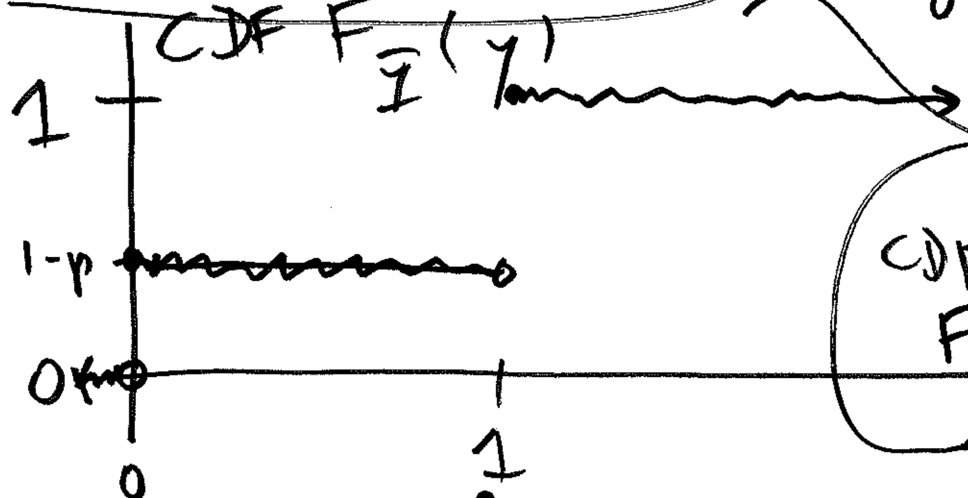


A: certainly: $Y \sim \text{Uniform}(a,b)$



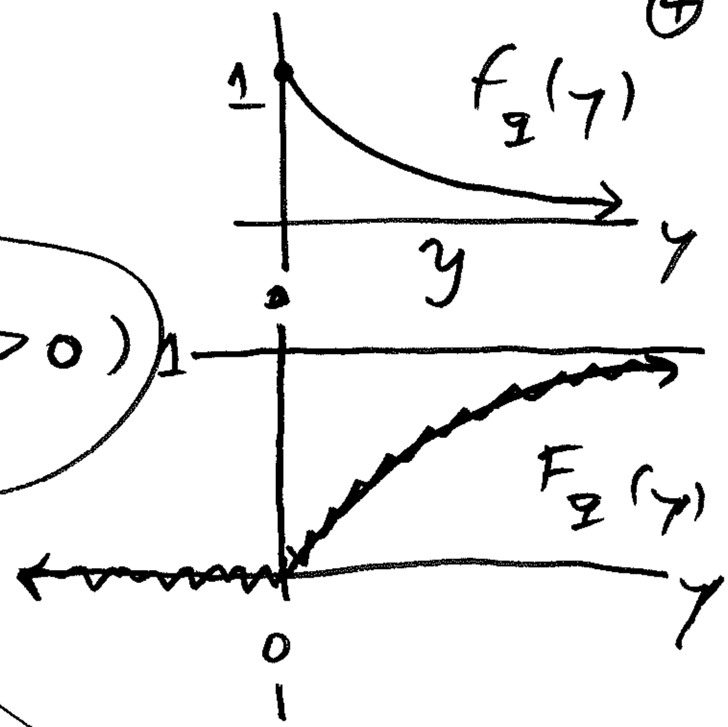
$f_Y(y) \sim \frac{1}{(1+y)^2}$



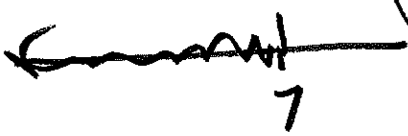


CDF F_Y step function $\leftrightarrow$ Y discrete rv voltage example

$$f_Y(y) = \frac{1}{(1+y)^2} I(y > 0)$$



CDF



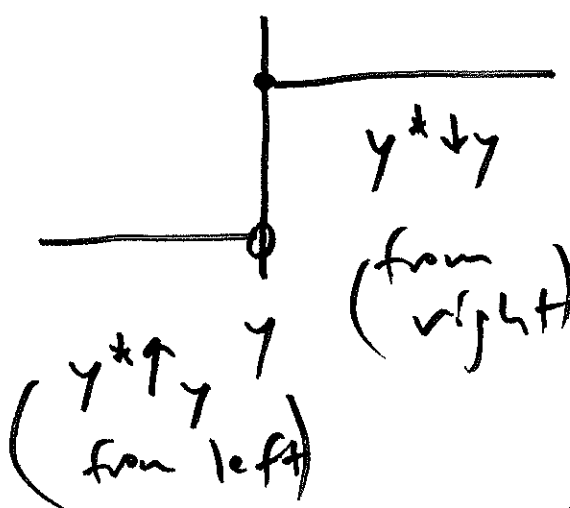
$$F_Y(y) = P(Y \le y)$$

PDF

$$= \int_{-\infty}^{y} f_Y(t)\, dt$$

PDF CDF

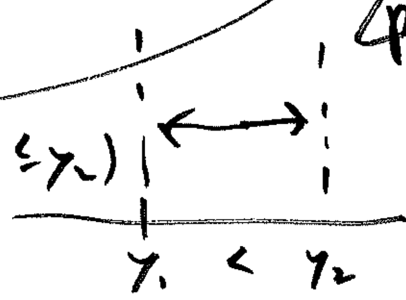
$$f_Y(y) = \frac{d}{dy} F_Y(y) = F_Y'(y)$$



$$F_Y(y) = P(\underline{Y \le y})$$

$$P(\underline{Y > y}) = 1 - F_Y(y)$$

$$F_Y(y_2) = P(Y \le y_2)$$



$$F_Y(y_1) = P(Y \le y_1)$$

10.49) $Y = (X \mid \lambda) \sim \text{Exponential}(\lambda)$

$$\leftrightarrow \underbrace{f_Y(y)}_{\text{PDF}} = \lambda e^{-\lambda y}\, I(y>0)$$

CDF $F_Y(y) = P(Y \le y) = \int_0^y \lambda e^{-\lambda t}\, dt$

(for $y>0$)

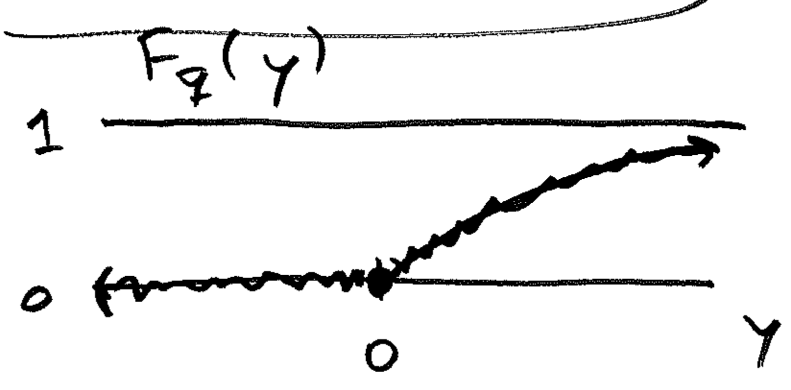
$$= \lambda \int_0^y e^{-\lambda t}\, dt = \cancel{\lambda}\left(\frac{e^{-\lambda t}}{-\cancel{\lambda}}\right)_0^y$$

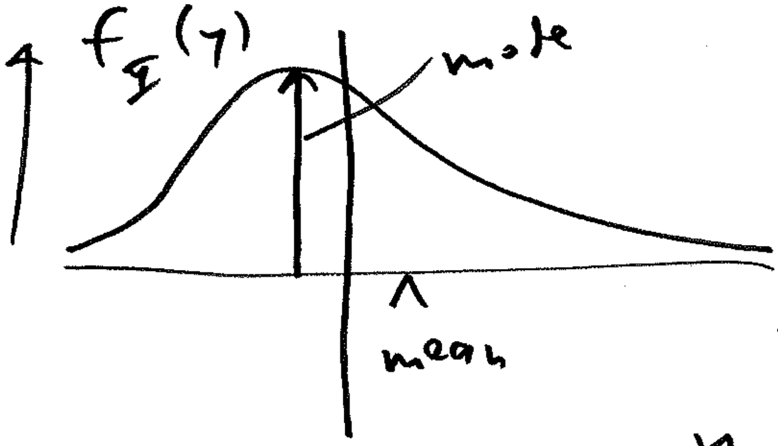
$$= -\left(e^{-\lambda y} - 1\right) = 1 - e^{-\lambda y}$$

CDF of Exponential(λ) distribution

$$F_Y(y) = \begin{cases} 0 & y \le 0 \\ 1 - e^{-\lambda y} & y > 0 \end{cases}$$

$$= \left(1 - e^{-\lambda y}\right) I(y>0)$$

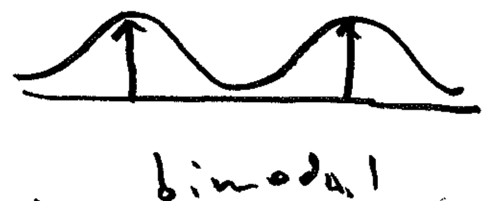


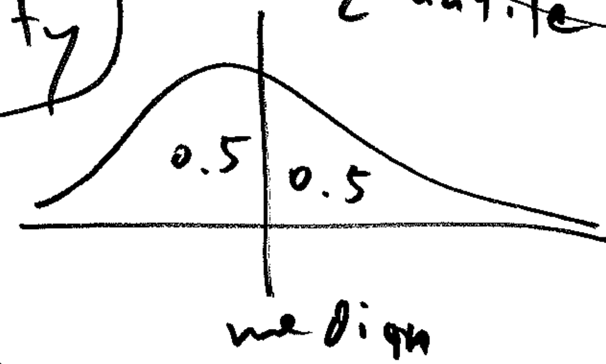


measures of center of unimodal dist. of Y?

median = 50th percentile = 0.5 quantile

50/50 point in probability





R Exponential distribution

dexp ← density (PDF)

pexp ← CDF

qexp ← inverse CDF (quantile function)

rexp ← random samples