

AMS 131: Quiz 5

Name: _____

You're working on a problem involving two continuous random variables X and Y , and you figure out that their joint PDF has the following form:

$$f_{X,Y}(x,y) = \begin{cases} cx^2 & \text{for } 0 \leq y \leq 1 - x^2 \\ 0 & \text{otherwise} \end{cases}. \quad (1)$$

(a) Sketch the support S of this bivariate distribution.

(b) Compute the normalizing constant c .

(c) It can be shown that the marginal PDFs of X and Y with this joint PDF are

$$f_X(x) = \begin{cases} \frac{15}{4}x^2(1-x^2) & \text{for } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

and

$$f_Y(y) = \begin{cases} \frac{5}{2}(1-y)^{\frac{3}{2}} & \text{for } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

Verify that both of these marginals are correct.

(d) Are X and Y independent in this joint distribution? Explain briefly.