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## AMS 131: Quiz 8

Name: $\qquad$

Someone offers you the possibility to play a gambling game with the following rules. First, you decide how much money you're willing to put at risk in this game: this amount let's call it $A$ - is referred to as your stake (all the monetary quantities are in dollars in this problem). Having chosen your stake, you're allowed to bet any amount $0 \leq B \leq A$ (thus, as a decision problem, for any fixed value of $A$, your possible actions in this situation correspond to values of $B$ ). If you win the bet, which occurs with probability $0<p<1$, your stake becomes $(A+B)$; if you lose, it becomes $(A-B)$, and this (of course) occurs with probability $(1-p)$; and (crucially) $p$ is known to you. Let $X$ denote the value of your stake after the gamble has occurred, and suppose that you agree with Daniel Bernoulli that a reasonable utility function is $U(x)=1+\log (x)$.
(a) Write out the probability mass function (PMF) for $X$.
(b) Work out your expected utility $E[U(X)]$ in this game, as a function of $A, B$ and $p$.
(c) Intuitively, what should you do (i.e., what value of $B$ should you choose) if $p<\frac{1}{2}$ ? Explain briefly.
(d) Show that when $p \geq \frac{1}{2}$ your expected utility is maximized with the choice $B=$ $(2 p-1) A$. Is this answer intuitively reasonable? Explain briefly.

